ISSN 2349-7815

International Journal of Recent Research in Electrical and Electronics Engineering (IJRREEE)
Vol. 9, Issue 2, pp: (11-16), Month: April - June 2022, Available at: www.paperpublications.org

Curvature of Plane Fractional Analytic Curve

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DOI: https://doi.org/10.5281/zenodo.6613839

Published Date: 04-June-2022

Abstract: **In this paper, based on Jumarie type of Riemann Liouville (R-L) fractional calculus, we mainly study the curvature of plane fractional analytic curve. A new multiplication of fractional analytic functions plays an important role in this article. Some examples are provided to illustrate our methods. In fact, these results we obtained are natural generalizations of those in traditional calculus.**

Keywords: **Jumarie type of R-L fractional calculus, curvature, plane fractional analytic curve, new multiplication, fractional analytic functions.**

I. INTRODUCTION

Fractional calculus deals with the derivatives and integrals of any real or complex order. In recent years, fractional calculus has been widely popularized and valued because of its applications in various fields such as mechanics, dynamics, elasticity, electronics, physics, modeling, economics, and control theory [1-8]. Fractional calculus is different from classical calculus. There is no unique definition of fractional derivative and integral. Commonly used definitions include Riemann Liouville (R-L) fractional derivative, Caputo fractional derivative, Grunwald Letnikov (G-L) fractional derivative, conformable fractional derivative, and Jumarie's modified R-L fractional derivative [9-11]. On the other hand, the application of fractional calculus in fractional differential equations can be referred to [12-14].

Based on Jumarie's modification of R-L fractional calculus, this paper studies how to calculate the curvature of plane fractional analytic curve. A new multiplication of fractional analytic functions plays an important role in this paper. We give two examples to illustrate our methods. In fact, the new multiplication is a natural operation of fractional analytic functions, and the results we obtained are generalizations of the results in classical calculus.

II. PRELIMINARIES

In this section, the fractional calculus used in this article and some properties are introduced.

Definition 2.1 ([15])**:** Let $0<\alpha\leq 1$, and x_0 be a real number. The Jumarie's modified Riemann-Liouville (R-L) α-fractional derivative is defined by

$$\left({}_{x_0}D_x^{\alpha}\right)[f(x)]=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_{x_0}^{x}\frac{f(t)-f(x_0)}{(x-t)^{\alpha}}dt. \tag{1}$$

And the Jumarie type of R-L α-fractional integral is defined by

$$\left({}_{x_0}I_x^{\alpha}\right)[f(x)]=\frac{1}{\Gamma(\alpha)}\int_{x_0}^{x}\frac{f(t)}{(x-t)^{1-\alpha}}dt. \tag{2}$$

Where $\Gamma(\ \ )$ is the gamma function. Furthermore, we define $\left({}_{x_0}D_x^{\alpha}\right)^n=\left({}_{x_0}D_x^{\alpha}\right)\left({}_{x_0}D_x^{\alpha}\right)\cdots\left({}_{x_0}D_x^{\alpha}\right)$ is the n-th order fractional derivative of $\left({}_{x_0}D_x^{\alpha}\right)$. We note that $\left({}_{x_0}D_x^{\alpha}\right)^n\neq{}_{x_0}D_x^{n\alpha}$ in general.

Proposition 2.2 ([16])**:** *Suppose that* α,β,x_0,C *are real numbers and* $\beta\geq\alpha>0$, *then*

$$\left({}_{x_0}D_x^{\alpha}\right)\left[(x-x_0)^{\beta}\right]=\frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}(x-x_0)^{\beta-\alpha}, \tag{3}$$

ISSN 2349-7815

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and

$$\left({}_{x_0}D_x^{\alpha}\right)[C]=0 \quad .$$

(4)

Next, we introduce the fractional analytic function.

Definition 2.3 ([17])**:** Let x, x_0, and a_k be real numbers for all k, $x_0 \in (a,b)$, and $0<\alpha\leq 1$. If the function $f_\alpha:[a,b]\to R$ can be expressed as $f_\alpha(x^\alpha)=\sum_{k=0}^{\infty}\frac{a_k}{\Gamma(k\alpha+1)}(x-x_0)^{k\alpha}$, an α-fractional power series on some open interval containing x_0, then we say that $f_\alpha(x^\alpha)$ is α-fractional analytic at x_0. Furthermore, if $f_\alpha:[a,b]\to R$ is continuous on closed interval $[a,b]$ and it is α-fractional analytic at every point in open interval (a,b), then f_α is called an α-fractional analytic function on $[a,b]$.

In the following, a new multiplication of fractional analytic functions is introduced.

Definition 2.4 ([18])**:** If $0<\alpha\leq 1$, and x_0 is a real number. Let $f_\alpha(x^\alpha)$ and $g_\alpha(x^\alpha)$ be two α-fractional analytic functions defined on an interval containing x_0 ,

$$f_\alpha(x^\alpha)=\sum_{k=0}^{\infty}\frac{a_k}{\Gamma(k\alpha+1)}(x-x_0)^{k\alpha}=\sum_{k=0}^{\infty}\frac{a_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}(x-x_0)^{\alpha}\right)^{\otimes k}, \tag{5}$$

$$g_\alpha(x^\alpha)=\sum_{k=0}^{\infty}\frac{b_k}{\Gamma(k\alpha+1)}(x-x_0)^{k\alpha}=\sum_{k=0}^{\infty}\frac{b_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}(x-x_0)^{\alpha}\right)^{\otimes k}. \tag{6}$$

Then we define

$$\begin{aligned}&f_\alpha(x^\alpha)\otimes g_\alpha(x^\alpha)\\&=\sum_{k=0}^{\infty}\frac{a_k}{\Gamma(k\alpha+1)}(x-x_0)^{k\alpha}\otimes\sum_{k=0}^{\infty}\frac{b_k}{\Gamma(k\alpha+1)}(x-x_0)^{k\alpha}\\&=\sum_{k=0}^{\infty}\frac{1}{\Gamma(k\alpha+1)}\left(\sum_{m=0}^{k}\binom{k}{m}a_{k-m}b_m\right)(x-x_0)^{k\alpha}.\end{aligned}$$

(7)

In other words,

$$\begin{aligned}&f_\alpha(x^\alpha)\otimes g_\alpha(x^\alpha)\\&=\sum_{k=0}^{\infty}\frac{a_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}(x-x_0)^{\alpha}\right)^{\otimes k}\otimes\sum_{k=0}^{\infty}\frac{b_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}(x-x_0)^{\alpha}\right)^{\otimes k}\\&=\sum_{k=0}^{\infty}\frac{1}{k!}\left(\sum_{m=0}^{k}\binom{k}{m}a_{k-m}b_m\right)\left(\frac{1}{\Gamma(\alpha+1)}(x-x_0)^{\alpha}\right)^{\otimes k} \quad .\end{aligned}$$

(8)

Definition 2.5 ([19])**:** Let $0<\alpha\leq 1$, and $f_\alpha(x^\alpha)$, $g_\alpha(x^\alpha)$ be two α-fractional analytic functions defined on an interval containing x_0 ,

$$f_\alpha(x^\alpha)=\sum_{k=0}^{\infty}\frac{a_k}{\Gamma(k\alpha+1)}(x-x_0)^{k\alpha}=\sum_{k=0}^{\infty}\frac{a_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}(x-x_0)^{\alpha}\right)^{\otimes k}, \tag{9}$$

$$g_\alpha(x^\alpha)=\sum_{k=0}^{\infty}\frac{b_k}{\Gamma(k\alpha+1)}(x-x_0)^{k\alpha}=\sum_{k=0}^{\infty}\frac{b_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}(x-x_0)^{\alpha}\right)^{\otimes k}. \tag{10}$$

The compositions of $f_\alpha(x^\alpha)$ and $g_\alpha(x^\alpha)$ are defined by

$$(f_\alpha\circ g_\alpha)(x^\alpha)=f_\alpha\left(g_\alpha(x^\alpha)\right)=\sum_{k=0}^{\infty}\frac{a_k}{k!}\left(g_\alpha(x^\alpha)\right)^{\otimes k}, \tag{11}$$

and

$$(g_\alpha\circ f_\alpha)(x^\alpha)=g_\alpha\left(f_\alpha(x^\alpha)\right)=\sum_{k=0}^{\infty}\frac{b_k}{k!}\left(f_\alpha(x^\alpha)\right)^{\otimes k}. \tag{12}$$

ISSN 2349-7815

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Definition 2.6 ([19])**:** Let $0 < \alpha \leq 1$. If $f_\alpha(x^\alpha)$, $g_\alpha(x^\alpha)$ are two α-fractional analytic functions at x_0 satisfies

$$(f_\alpha \circ g_\alpha)(x^\alpha) = (g_\alpha \circ f_\alpha)(x^\alpha) = \frac{1}{\Gamma(\alpha+1)}(x - x_0)^\alpha. \tag{13}$$

Then $f_\alpha(x^\alpha)$, $g_\alpha(x^\alpha)$ are called inverse functions of each other.

The followings are some fractional analytic functions.

Definition 2.7([20])**:** If $0 < \alpha \leq 1$, and x is a real number. The α-fractional exponential function is defined by

$$E_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{x^{k\alpha}}{\Gamma(k\alpha+1)} = \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes k}. \tag{14}$$

And the α-fractional logarithmic function $Ln_\alpha(x^\alpha)$ is the inverse function of $E_\alpha(x^\alpha)$. In addition, the α-fractional cosine and sine function are defined respectively as follows:

$$cos_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k\alpha}}{\Gamma(2k\alpha+1)} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes 2k}, \tag{15}$$

and

$$sin_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{(2k+1)\alpha}}{\Gamma((2k+1)\alpha+1)} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes(2k+1)}. \tag{16}$$

In the following, the power of fractional analytic function is introduced.

Definition 2.8 [21]**:** Let $0 < \alpha \leq 1$, and r be a real number. The r-th power of the α-fractional analytic function $f_\alpha(x^\alpha)$ is defined by $[f_\alpha(x^\alpha)]^{\otimes r} = E_\alpha\left(r \cdot Ln_\alpha\left(f_\alpha(x^\alpha)\right)\right)$.

Theorem 2.9 ([22])**:** *Let* $0 < \alpha \leq 1$, *and* x *be a real number, then*

$$[sin_\alpha(x^\alpha)]^{\otimes 2} + [cos_\alpha(x^\alpha)]^{\otimes 2} = 1, \tag{17}$$

Definition 2.10: The smallest positive real number T_α such that $E_\alpha(iT_\alpha) = 1$, is called the period of $E_\alpha(ix^\alpha)$.

III. RESULTS AND EXAMPLES

In the following, the definition of curvature of parametric plane fractional analytic curve is provided.

Definition 3.1: Let $0 < \alpha \leq 1$. If the parametric equation of plane α-fractional analytic curve is

$$\begin{cases} x_\alpha = x_\alpha(t^\alpha), \\ y_\alpha = y_\alpha(t^\alpha). \end{cases} \tag{18}$$

Where $x_\alpha(t^\alpha)$ and $y_\alpha(t^\alpha)$ are α-fractional analytic functions at $t = t_0$. Then the curvature of this curve is defined by

$$K = \left| \begin{array}{c} \left[\left({}_{t_0}D_t^\alpha\right)[x_\alpha(t^\alpha)] + A\right] \otimes \left[\left({}_{t_0}D_t^\alpha\right)^2[y_\alpha(t^\alpha)] + B\right] - \left[\left({}_{t_0}D_t^\alpha\right)^2[x_\alpha(t^\alpha)] + C\right] \otimes \left[\left({}_{t_0}D_t^\alpha\right)[y_\alpha(t^\alpha)] + D\right] \\ \otimes \left[\left(\left({}_{t_0}D_t^\alpha\right)[x_\alpha(t^\alpha)] + A\right)^{\otimes 2} + \left(\left({}_{t_0}D_t^\alpha\right)[y_\alpha(t^\alpha)] + D\right)^{\otimes 2}\right]^{\otimes\left(-\frac{3}{2}\right)} \end{array} \right|. \tag{19}$$

Where A, B, C, D are constants such that

$$\begin{cases} \left({}_{t_0}D_t^1\right)[x_1(t)] + A = \frac{d}{dt}x_1(t), \\ \left({}_{t_0}D_t^1\right)^2[y_1(t)] + B = \frac{d^2}{dt^2}y_1(t), \\ \left({}_{t_0}D_t^1\right)^2[x_1(t)] + C = \frac{d^2}{dt^2}x_1(t), \\ \left({}_{t_0}D_t^1\right)[y_1(t)] + D = \frac{d}{dt}y_1(t). \end{cases} \tag{20}$$

ISSN 2349-7815

International Journal of Recent Research in Electrical and Electronics Engineering (IJRREEE)
Vol. 9, Issue 2, pp: (11-16), Month: April - June 2022, Available at: www.paperpublications.org

Remark 3.2: If $\alpha = 1$, then the curvature of this curve becomes the curvature of classical parametric plane analytic curve.

Next, we give the formula of the curvature of plane fractional analytic function.

Theorem 3.3: *Suppose that* $0 < \alpha \leq 1$. *Then the curvature of* α*-fractional analytic function* $y_\alpha = y_\alpha(x^\alpha)$ *is*

$$K = \left| \left[\left({}_{x_0}D_x^\alpha \right)^2 [y_\alpha(x^\alpha)] + E \right] \otimes \left[1 + \left(\left({}_{x_0}D_x^\alpha \right) [y_\alpha(x^\alpha)] + F \right)^{\otimes 2} \right]^{\otimes\left(-\frac{3}{2}\right)} \right|. \tag{21}$$

Where E, F are constants such that

$$\begin{cases} \left({}_{x_0}D_x^1 \right)^2 [y_1(x)] + E = \frac{d^2}{dx^2} y_1(x), \\ \left({}_{x_0}D_x^1 \right) [y_1(x)] + F = \frac{d}{dx} y_1(x). \end{cases} \tag{22}$$

Proof The α-fractional analytic function $y_\alpha = y_\alpha(x^\alpha)$ can be expressed as

$$\begin{cases} x_\alpha = \frac{1}{\Gamma(\alpha+1)} t^\alpha, \\ y_\alpha = y_\alpha(t^\alpha). \end{cases} \tag{23}$$

Therefore, by Definition 3.1, we obtain the curvature of this curve

$$K = \left| \left[\left({}_{x_0}D_x^\alpha \right)^2 [y_\alpha(x^\alpha)] + E \right] \otimes \left[1 + \left(\left({}_{x_0}D_x^\alpha \right) [y_\alpha(x^\alpha)] + F \right)^{\otimes 2} \right]^{\otimes\left(-\frac{3}{2}\right)} \right|.$$ Q.e.d.

Remark 3.4: If $\alpha = 1$, then the curvature of this curve is that of classical analytic function.

In the following, we provide two examples to illustrate how to calculate the curvature.

Example 3.5: Assume that $0 < \alpha \leq 1$, and $r, s > 0$. Find the curvature of parametric α-fractional elliptic curve

$$\begin{cases} x_\alpha(t^\alpha) = r \cdot cos_\alpha(t^\alpha), \\ y_\alpha(t^\alpha) = s \cdot sin_\alpha(t^\alpha). \end{cases} \tag{24}$$

Where $0 \leq t \leq (T_\alpha)^{\frac{1}{\alpha}}$.

Solution By Definition 3.1, we obtain the curvature of this curve

$$K = \left| \begin{array}{c} \left({}_0D_t^\alpha \right)[r \cdot cos_\alpha(t^\alpha)] \otimes \left({}_0D_t^\alpha \right)^2 [s \cdot sin_\alpha(t^\alpha)] - \left[\left({}_0D_t^\alpha \right)^2 [r \cdot cos_\alpha(t^\alpha)] + r \right] \otimes \left[\left({}_0D_t^\alpha \right)[s \cdot sin_\alpha(t^\alpha)] + s \right] \\ \otimes \left[\left(\left({}_0D_t^\alpha \right)[r \cdot cos_\alpha(t^\alpha)] \right)^{\otimes 2} + \left(\left({}_0D_t^\alpha \right)[s \cdot sin_\alpha(t^\alpha)] + s \right)^{\otimes 2} \right]^{\otimes\left(-\frac{3}{2}\right)} \end{array} \right|$$

$$= \left| [r \cdot sin_\alpha(t^\alpha) \otimes s \cdot sin_\alpha(t^\alpha) + r \cdot cos_\alpha(t^\alpha) \otimes s \cdot cos_\alpha(t^\alpha)] \otimes [r^2 \cdot [sin_\alpha(t^\alpha)]^{\otimes 2} + s^2 \cdot [cos_\alpha(t^\alpha)]^{\otimes 2}]^{\otimes\left(-\frac{3}{2}\right)} \right|$$

$$= rs \left| [r^2 \cdot [sin_\alpha(t^\alpha)]^{\otimes 2} + s^2 \cdot [cos_\alpha(t^\alpha)]^{\otimes 2}]^{\otimes\left(-\frac{3}{2}\right)} \right|.$$

(25)

Example 3.6: Let $0 < \alpha \leq 1$, and $p \neq 0$. Evaluate the curvature of the α-fractional analytic curve

$$y_\alpha(x^\alpha) = p \left[1 + \frac{1}{\Gamma(\alpha+1)} x^\alpha \right]^{\otimes(-1)}. \tag{26}$$

Where $x \geq 0$.

Solution Using Theorem 3.3 yields the curvature of this curve is

$$K = \left| \left[\left({}_0D_x^\alpha \right)^2 \left[p \left[1 + \frac{1}{\Gamma(\alpha+1)} x^\alpha \right]^{\otimes(-1)} \right] \right] \otimes \left[1 + \left(\left({}_0D_x^\alpha \right) \left[p \left[1 + \frac{1}{\Gamma(\alpha+1)} x^\alpha \right]^{\otimes(-1)} \right] \right)^{\otimes 2} \right]^{\otimes\left(-\frac{3}{2}\right)} \right|$$

$$= \left| 2p \cdot \left[1 + \frac{1}{\Gamma(\alpha+1)} x^{\alpha}\right]^{\otimes(-3)} \otimes \left[1 + \left(-p\left[1 + \frac{1}{\Gamma(\alpha+1)} x^{\alpha}\right]^{\otimes(-2)}\right)^{\otimes 2}\right]^{\otimes\left(-\frac{3}{2}\right)} \right|$$

$$= \left| 2p \cdot \left[1 + \frac{1}{\Gamma(\alpha+1)} x^{\alpha}\right]^{\otimes(-3)} \otimes \left[1 + p^2\left[1 + \frac{1}{\Gamma(\alpha+1)} x^{\alpha}\right]^{\otimes(-4)}\right]^{\otimes\left(-\frac{3}{2}\right)} \right|. \tag{27}$$

IV. CONCLUSION

The purpose of this paper is to study how to evaluate the curvature of plane fractional analytic curve based on Jumarie's modified R-L fractional calculus. In addition, a new multiplication plays an important role in this article. The results we obtained are generalizations of those in classical calculus. In fact, the new multiplication we defined is a natural operation of fractional analytic functions. In the future, we will continue to use Jumarie type of R-L fractional calculus and the new multiplication to study problems in engineering mathematics and fractional differential equations.

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ISSN 2349-7815

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