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A JOURNEY INTO MATHEMATICAL JURIMETRICS

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[1] This Study is derived from Nnamuchi O.E's P.hD Thesis Study – A Journey into Mathematical Jurimetrics

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Abstract: **Mathematics is an art of giving the same name to different things (Henri Poincare). Properties of commonly occurring physical and abstract spaces are often generalized using Metric Spaces. This is by building some Mathematical framework into them. This work is one in a series of papers derived from the doctoral dissertation which was based on the formalization of Metric Topological Space in the milieu of Legal Jurisprudence; a Metric Topological Space (called MTS) in which the points are bodies of evidence with the concept of nearness of points defining the evidence clearly stipulated, thus determining an earliest systematic and Mathematical approach to *Jurimetrics* in the recent times.**

Keywords: **Mathematical framework, *Jurimetrics*, Metric Topological Space (called MTS),.**

1. INTRODUCTION

1.1 Background of the study

Searching has always been one of the most prominent data processing operations because of its usefulness in delivering required information efficiently [1, 2]. However, exact match retrieval, typical for traditional databases, is not sufficient or feasible for data types of present digital age, that is, image databases, text documents, audio and video collections, DNA or protein sequences, to name a few [3]. What seems to be more useful, if not necessary, is to base the search paradigm at any given time on a form of *proximity*, *similarity*, or *dissimilarity* of a query and data object. Roughly speaking, objects that are close to a given query object form the query response set. This similarity is often measured by a certain distance function [4]. In this regard, the notion of mathematical *metric spaces* provides a useful abstraction for the nearness [5].

Consider this scenario: Ordinary physical space, with distances measured as by a ruler, which is, is so due to the following.

(i) There are points

(ii) There is a distance between any two given points

(iii) All the distances are nonnegative

(iv) Distance is the same in either direction

(v) Two points have a zero (0) distance between them if and only if they are the same point, and

(vi) The triangle inequality (the shortest distance between any two points is the direct distance), applies.

1.1.1 Domesticating the above concepts of a Metric Space in a jurimetric Space

All the description from (i) – (vi) are true for a *jurimetric Space* whch is defined below in Definition 1.2. What suffices is to establish these using practical analysis of selected Case Laws. That is, It can be seen that:

(i) there exist points (Case laws),

(ii) there is a distance (Similarity or dissimilarity of Case Laws) between any two points in law, constituting a measure of similarity and dissimilarity,

(iii) all the distances are nonnegative,

(iv) distance is the same in either direction (Similarity of Case ***A*** to ***B*** is same as Similarity of Case ***B*** to ***A***),

(v) two points have distance zero *iff* they are the same point (in the perspectives of this research, a Case Law cannot be compared to itself). The result of comparing a Case Law to itself is 0, the converse is also true, the implication of the "iff" for which one side has been addressed, the other is that if the distance between two points is 0 then both points are necesssarily the same or belong to the same equivalence class of a suitable defined relation; that is, if the similarity between case ***A*** and case ***B*** is 0, then case ***A*** is the same as case ***B*** or they both belong to an equivalence class of cases based on a well defined relation on the set of cases), and

(vi) the triangle inequality applies (This would have ordinarily implied that there is a closer similarity between one case and another than there is similarity between that case and a third case), however the proper position of the triangle inequality is that similarity between case ***A*** and case ***B*** is at most equal to the similarity between case ***A*** and case ***C*** plus the similarity between case ***B*** and case ***C*** for any other case ***C*** **).** Put in another perspective, it is easier to determine similarity between case ***A*** and Case ***B*** than it is to determine similarity between the said case ***A*** and another case ***C*** to which similarity between it and ***B*** has been established.

1.1.2 A Basis for Mathematical Jurimetrics

Jurisprudence among other things seeks to show the relationship between law and other values, such as, morality and justice [6, p8]. Jurisprudence is known as the Science of Law. The Case Laws may define an abstract space of the kind we are studying as many kinds of space exist that generalise in different ways some physical space of human reality.

1.1.3 What is Jurisprudence?

Jurisprudence comes from the Latin term *juris prudentia,* which means, the study, knowledge, or science of law [7]. That signifies like any other social study, law can also be studied scientifically or systematically. In modern law, jurisprudence is understood as a term that embraces spectrum of questions about the nature and purpose of law and responses made to them. Jurisprudence has many aspects, with four types being the most common. The most prevalent form of jurisprudence is that it seeks to analyze, explain, classify, and criticize entire bodies of law, ranging from contract to tort to constitutional law. Legal encyclopedias, law reviews, and law school textbooks frequently contain this type of jurisprudential scholarship.

The second type of jurisprudence compares and/or contrasts law with other fields of knowledge such as literature, economics, religion, the social sciences, Philosophy, Mathematics etc. The purpose of this interdisciplinary study is to enlighten each field of knowledge by sharing insights that have proved important to understanding essential features of the comparative disciplines. The third type of jurisprudence raises fundamental questions about the law itself. These questions seek to reveal the historical, moral, and cultural underpinnings of a particular legal concept. *The Common Law* (1881), written by Oliver Wendell Holmes, Jr., is a well - known example of this type of jurisprudence [7]. It traces the evolution of civil and criminal responsibility from undeveloped societies where liability for injuries was based on subjective notions of revenge, to modern societies where liability is based on objective notions of reasonableness. The fourth and fastest-growing body of jurisprudence focuses on even more abstract questions, including, what is law? What is its relation to justice and morality? What is the role of a judge? Is a judge more like a legislator who simply decides a case in favor of the most politically preferable outcome? What is justice? What is liberty and freedom? Our aim in this dissertation is to treat jurisprudence in the sense of the second type of Jurisprudence [6, p11].

1.2. Why Jurisprudence AND Mathematics

At the practical level, reading and participating in jurisprudence is primarily concerned with application of the concept of metric spaces and their properties to the legal arena of jurisprudence. Specifically, the focus is to create a metric space where the points in the space are law cases and the distance between any two points (law cases) in the space is some measure of nearness or farness (similarity or dissimilarity) of any two cases.

1.2.1 Mathematically Finding Similar Judgments

The problem of finding similar judgments, as in this research work, is to find a set of judgments (Case Laws), C_s which are similar to a given judgment (Case Law) *J*. Therefore a new Metric Space, hereinafter referred to as Metric Case Space (*MCS*) can be defined on the body (bodies) of evidence. In this case the evidence is/are the case law (s) or otherwise in the

body of judgment J or set of judgments similar to judgment J, it is referred to as the 'Act', which is the stipulated law employed to enable the judge take his decisions on the matter present. Therefore, it is clear that finding similar Judgments is comparable to the notion of *distance function*. **A distance function is a Mathematical Formula used by distance Metrics** which together with a *Metric* define a Metric Space. So we now define this new space called *Jurimetric Space* as (J, C_s), and denote by C_s, the set of all cases similar to the case J.

Definition 1.1: (Following the definition of a Metric Space)

Let J and $C_s : C_i \times C_i \rightarrow R$ be a function that may or may not satisfy these properties, for all $C_i, C_j, C_k \in C_s$:

$$C_s(C_i, C_j) \geq 0 \quad (1.1)$$

$$C_s(C_i, C_i) = 0 \quad (1.2)$$

$$C_s(C_i, C_j) \neq 0 \text{ if } C_i \neq C_j \quad (1.3)$$

$$C_s(C_i, C_j) = C_s(C_j, C_i) \quad (1.4)$$

$$C_s(C_i, C_k) \leq C_s(C_i, C_j) + C_s(C_j, C_k) \quad (1.5)$$

Property (1.5), which is of particular importance to our work, is called the triangular inequality for a Metric Case Space. It is of particular importance because as in Metric Spaces, it is a requiremnet upon distance, and is responsible for most of the interesting structure we might need to construct, namely convergence.

Definition 1.2: In general, any function say $\boldsymbol{d: S \times S \rightarrow R}$ used to express some concept of distance is called a *distance function* and has the mathematical properties which we have used in defining the new Space called Jurimetric Space (J, C_s). In d, elements of the set S are called *points*. The pair (S, d) is called a *metric space*. So elements of C_s are as well points in C_s called *juristance function*, thus defining the pair (J, Cs) called Juri*metric Space*. We define other kinds of Jurimetrics by relaxing one or more of the properties:

Property 1.2.1

In definition 1.1, without (1.3), it is a *pseudojurimetric*;

Property 1.2.2

In definition 1.1, without (1.4), we have a *quasijurimetric*;

Property 1.2.3

In definition 1.1, without (1.5), we have a *semijurimetric*; and without any of those (leaving only (1.1) and (1.2)), the metric is a *prajurimetric* [9].

$$Where\ C_s = \{ C_i, i > 0, i \in N \},$$

and C_i represents the judgements (Case Laws) in the set C_s which are similar to J. The following properties of a Metric Space (X, d) is traceable to a Metric Case Space (J, C_s).

(i) $C_s(C_i, C_j) = C_s(C_j, C_i)$, We call this the symmetric property. It establishes that their similarity is the same whichever case comes first in the set of cases in the case space, taking any two at a time.

(ii) $C_s(C_i, C_j) \neq 0 \Rightarrow C_i \neq C_j$. The contrapositive of $C_s(C_i, C_j) = 0 \Rightarrow C_i = C_j$ is $C_i \neq C_j \Rightarrow C_s(C_i, C_j) \neq 0$. Similarity of case Laws does not imply equality in Case Laws. That is to say Case laws need not be equal to be similar, otherwise similarity fails. Similarity is checked on the attributes of the case Law (s) being reviewed in time.

Definition 1.3: Common law is defined as a body of legal rules that have been made by judges as they issue rulings on cases, as opposed to rules and laws made by the legislature or in official statutes (It can simply be understood as Case Laws).

Definition 1.4: Attributes of a Case Law - The following define attributes of a Case Law:

A case Law has the following attributes or components:

The components or attributes of a typical judgement (Case Law) are as shown below in 1.4.1 - 1.4.6 that follow.

1.4.1 Name of judgment: Name of judgment is given as per the name of Appellant's and defendant's name. In Appendix 1, name of the judgment is "Khandesh spg & wvg mills co. ltd. V. The Rashtriya Girni Kamgar Sangh Jalagaon".

1.4.2 Names of judges: Names of those judges who delivered the judgment after hearing the case are mentioned under this section. Fo r example, the judgment presented in Appendix 1 is delivered by three judges bench of Supreme court of India and name of judges are "K Subbarao", "P.B Gajendragadkar" and "K.C. Das Gupta".

1.4.3 Unique Identifier/Reference: Judgement contains unique identifiers through which this judgment will be referred by other judgments. Format of these names vary according to law reporters. In general, the format contain: title of the reports, volume number, page number and year (of publication). For example, (1988) 2 SCR 809 - where '1988' corresponds to year of publication, '2' corresponds to volume of the reporter, 'SCR' corresponds to name of the reporter (abbreviation of Supreme Court Reporter) and 809 is page Number of the judgment within the volume. In Appendix 1, the judgment contains two different identifiers. These IDs are, 1960 AIR 571, 1960 SCR(2) 841.

1.4.4 Act: This categorizes the issue discussed in the judgment from legal point of view. Since, a judgment resolves in a dispute between two or more parties involved, Act pertains all legal specification of the matter involved in the dispute. For example, in Appendix 1, "Industrial dispute-bonus-full bench formula-rehabilitation-reserves used as working capital-mode of proof".

1.4.5 Headnote: Headnote is a summary of the text of a court decision to aid readers. Generally, a legal judgment may be very big in size due to which, it is quite difficult to read the whole judgment. To make a judgment easier to analyze, summary of the judgment is prepared which is known as headnote.

1.4.6 Citation: These are embedded into the headnote text of the judgments. It represents older legal judgments which are referred for pronouncing the current judgment. In Appendix 1, [1960] 2 S.C.R 32, and [1960] 1 S.C.R 1 are the citations. [10]

Theorem 1.0: Two Case Laws can only be equal if they are exactly same in all attributes. In which case one can only say $C_i = C_i$ or $C_j = C_j$ which is Mathematical tautology.

Corollary 1.0: *If two judgments* (Case Laws) *cite the same set of judgments* (Case Laws), *both agree to the context of the cited judgments* (Case laws). What is the import of this assertion? To enable formulation of the corollary just identified therewith.

(iii) Triangular Inequality

$$C_s(C_i, C_k) \leq C_s(C_i, C_j) + C_s(C_i, C_k)$$

Theorem 1.1: Let J be a given judgement (Case Law) and (C_i, C_j) and (C_i, C_k) be pairs of Case Laws Similar to J. Then the common laws (case laws) (C_i, C_j) and (C_i, C_k) are respectively symmetric.

We have seen from the above definition of Jurimetric Space that $C_s(C_i, C_j) = C_s(C_j, C_i)$, recall that this was deduced for Case Laws from the Symmetric Property of Metric Spaces. It thus implies that similarity is the same whichever case comes first in the set of cases in the Metric Case Space (*MCS*). That is, Metric Case Space is Symmetric.

Corollary 1.1: The sum of any two Jurimetric Space is obviously symmetric.

Observation has revealed that various courts have different formats of judgments. Therefore case laws from the same source is likely to produce similar judgments [1, 2]**.** This has informed the reason for this research work using case Laws from Supreme Court, the Nigerian, Apex Court.

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